Physics: Introduction to Electromagnetic theory
Subject code: BSC-PHY-101G

ECE/ME

Ist Semester

Introduction of subject

- Electromagnetic Theory covers the basic principles of electromagnetism:
  - experimental basis, electrostatics, magnetic fields of steady currents, motional e.m.f. and electromagnetic induction, Maxwell's equations, propagation and radiation of electromagnetic waves, electric and magnetic properties of matter, and conservation laws.
  - This is a graduate level subject which uses appropriate mathematics but whose emphasis is on physical phenomena and principles.

Syllabus

UNIT – I: Electrostatics in vacuum and linear dielectric medium

Calculation of electric field and electrostatic potential for a charge distribution; Divergence and curl of electrostatic field; Laplace's and Poisson's equations for electrostatic potential Boundary conditions of electric field and electrostatic potential; energy of a charge distribution and its expression in terms of electric field. Electrostatic field and potential of a dipole. Bound charges due to electric polarization; Electric displacement; boundary conditions on displacement.

UNIT – II: Magnetostatics

Bio-Savart law, Divergence and curl of static magnetic field; vector potential and calculating It for a given magnetic field using Stokes' theorem; the equation for the vector potential and its solution for given current densities. Magnetostatics Ina linear magnetic medium: Magnetization and associated bound currents; auxiliary magnetic field; Boundary conditions on B and H. Solving for magnetic field due to simple magnets like a bar magnet; magnetic susceptibility and ferromagnetic, paramagnetic and diamagnetic materials.

UNIT – III: Faraday's law and Maxwell's equations

Faraday's law in terms of EMF produced by changing magnetic flux; equivalence of Faraday's law and motional EMF; Lenz's law; Electromagnetic breaking and its applications; Differential form of Faraday's law; energy stored in a magnetic field.Continuity equation for current densities; Modified equation for the curl of magnetic field to satisfy continuity, equation; displacement current and magnetic field arising from time-dependent electric field; Maxwell's equation in vacuum and non-conducting medium; Energy in an electromagnetic field; Flow of energy and Poynting vector.

UNIT – IV: Electromagnetic waves

The wave equation; Plane electromagnetic waves in vacuum, their transverse nature and polarization; relation, between electric and magnetic fields of an electromagnetic wave; energy carried by electromagnetic waves and, examples. Momentum carried by electromagnetic waves and resultant pressure. Reflection and transmission of electromagnetic waves from a non-conducting medium-vacuum interface for normal incidence.

Future scope

In a vast populated country like India, the requirement of energy is never ending. In recent years the country has witnessed huge requirement of electric energy and this has created numerous opportunities such as

- Bajaj International Private Ltd
- Crompton Greaves Limited
- ABB
- Reliance Power Ltd
- ONGS (Oil and Natural Gas Corporation)
- PGCIL (Power Grid Corporation of India Limited)
- CIL (Coal India Limited)
- BHEL (Bharat Heavy Electricals Limited)
- SAIL (Steel Authority of India Limited)
- Siemens Ltd
- Wipro Lighting Ltd

Unit 1 : Electrostatics in vacuum and linear dielectric medium

TABLE OF CONTENTS

Electric Field

- An **electric field** is said to exist in the region of space around a charged object. This charged object is the **source charge**
- When another charged object, the **test charge**, enters this electric field, an electric force acts on it.
- The electric field is defined as the electric force on the test charge per unit charge.

Electric Field: Vector Form

- From Coulomb's law, force between the source and test charges, can be expressed as

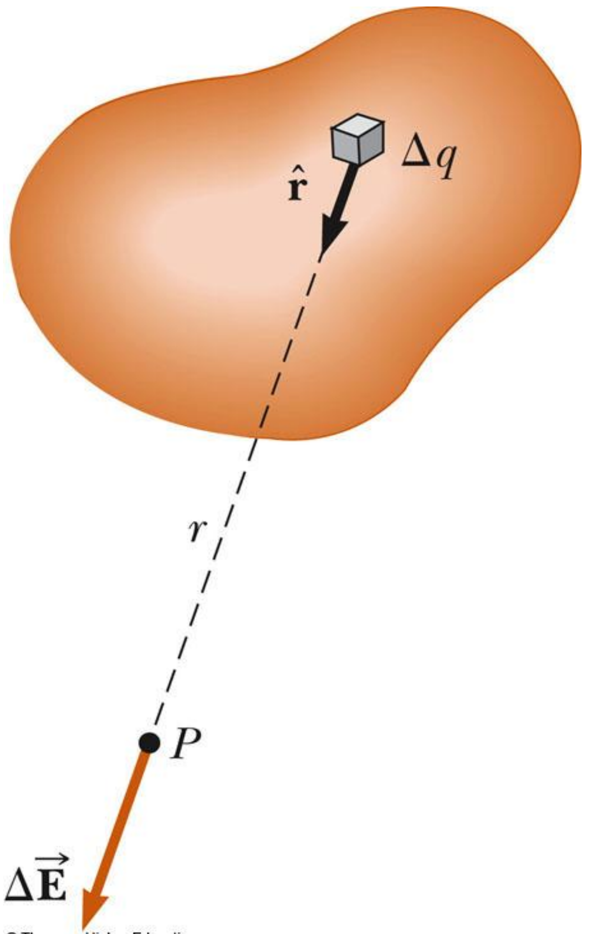
$$\vec{\mathbf{F}}_e = k_e \frac{q q_o}{r^2} \hat{\mathbf{r}}$$

- Then, the electric field will be

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}_e}{q_o} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

Electric Field – Continuous Charge Distribution

- Point charge – charge with zero size
- Continuous charge – object with charge distribution



- For the individual charge elements

$$\Delta\vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}}$$

- Because the charge distribution is continuous

$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}$$

Charge distribution

- If the charge is uniformly distributed over a volume, surface, or line, the amount of charge, dq, is given by
  - For the volume: $dq = \rho\, dV$
  - For the surface: $dq = \sigma\, dA$
  - For the length element: $dq = \lambda\, d\ell$

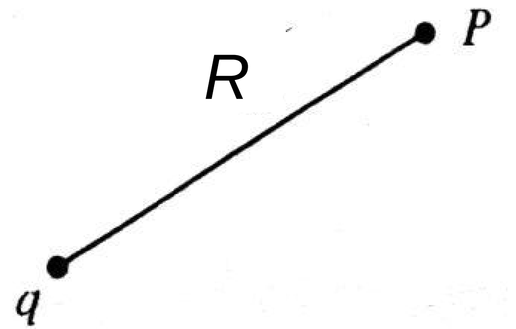
Charge Densities

- **Volume charge density**: when a charge is distributed evenly throughout a volume
  - $\rho \equiv Q / V$ with units C/m^3
- **Surface charge density**: when a charge is distributed evenly over a surface area
  - $\sigma \equiv Q / A$ with units C/m^2
- **Linear charge density**: when a charge is distributed along a line
  - $\lambda \equiv Q / \ell$ with units C/m

Charge Distribution of electric potential

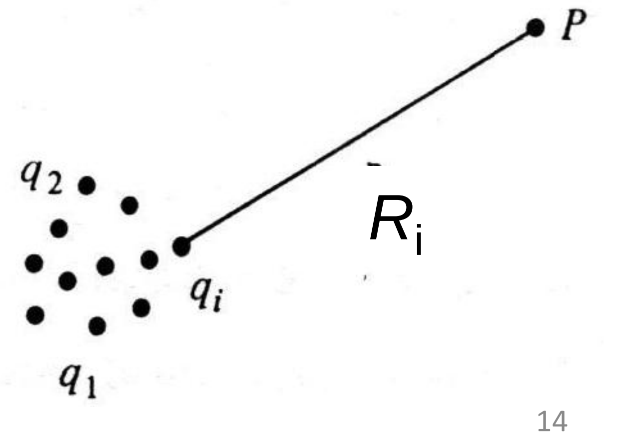
- Potential for a point charge

$$V(P) = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \qquad R = \left|\vec{r} - \vec{r}_p\right|$$



- Potential for a collection of charge

$$V(P) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{R_i} \qquad R_i = \left|\vec{r}_i - \vec{r}_p\right|$$



- Potential of a continuous distribution

for volume charge

$$\delta q = \rho d\tau$$

for a line charge

$$\delta q = \lambda d\ell$$

for a surface charge

$$\delta q = \sigma da$$

$$V(P) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho}{R} d\tau \qquad V(P) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda}{R} d\ell \qquad V(P) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma}{R} da$$

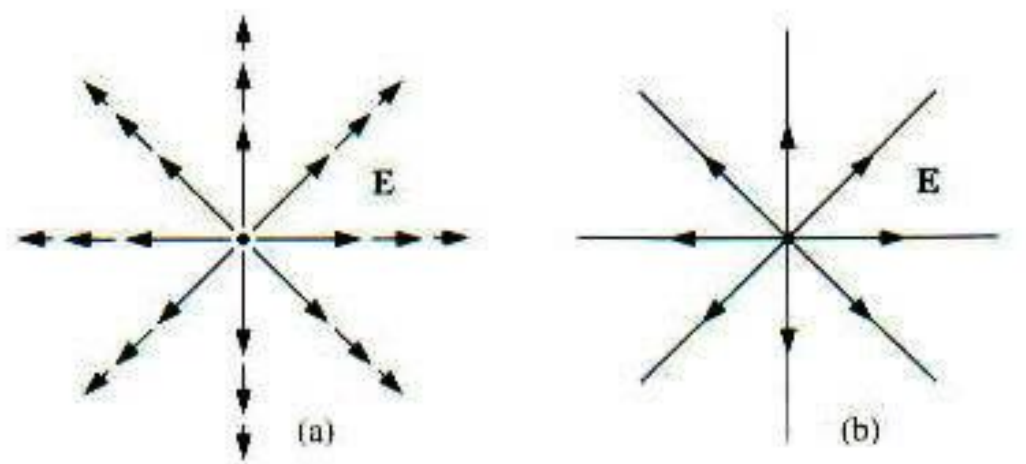
- **Corresponding electric field**

$$\vec{E}(P) = \frac{1}{4\pi\varepsilon_0}\int \frac{\hat{r}}{R^2}\rho d\tau \qquad \vec{E}(P) = \frac{1}{4\pi\varepsilon_0}\int \frac{\hat{r}}{R^2}\lambda d\ell \qquad \vec{E}(P) = \frac{1}{4\pi\varepsilon_0}\int \frac{\hat{r}}{R^2}\sigma\, da$$

Divergence and Curl of Electrostatic Fields

- Field of a point charge

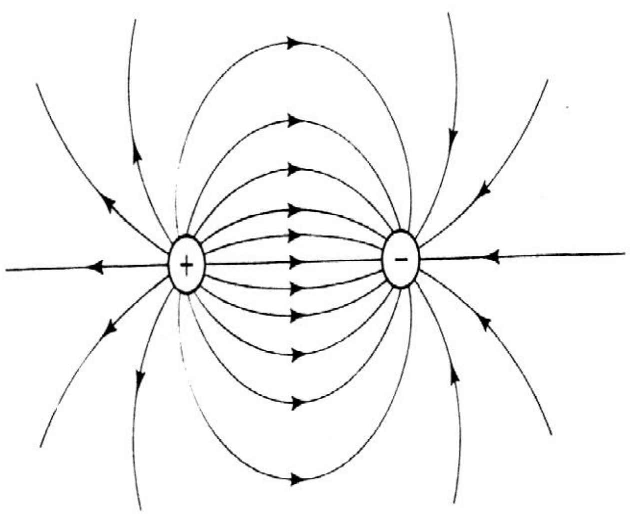
$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$



arrows   field lines: direction of $\mathbf{E}$

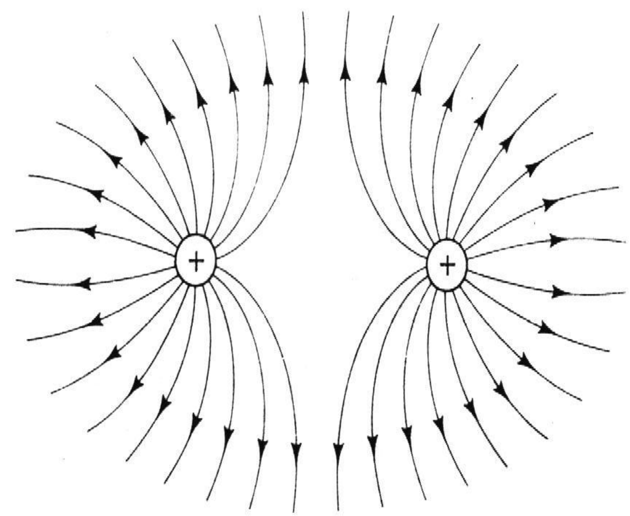
number of lines/area $\propto E$

1. Field lines emanate from a point charge symmetrically in all directions.
2. Field lines originate on positive charges and terminate on negative ones.
3. They cannot simply stop in midair, though they may extend out to infinity.
4. Field lines can never cross.



Equal but opposite charges

Figure 2.14



Equal charges

Figure 2.15

Gauss Law

- Gauss's Law in integral form $\oint \vec{E} \cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}$

Turn integral form into a differential one , by applying the divergence theorem

$$\oint_{surface} \vec{E} \cdot d\vec{a} = \int_{volume} (\nabla \cdot \vec{E}) d\tau$$

$$= \frac{1}{\varepsilon_0} Q_{enc} = \int_{volume} (\frac{1}{\varepsilon_0} \rho) d\tau$$

Gauss's law in differential form $\nabla \cdot \vec{E} = \frac{1}{\varepsilon_0} \rho$

The Divergence of E

- Calculate the divergence of E directly

$$\nabla \cdot \vec{E} = \frac{1}{4\pi\varepsilon_0} \int \nabla \cdot (\frac{\hat{R}}{R^2}) \rho(r') d\tau$$

The r-dependence is contained in R

$$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0} \int_{all\, space} \frac{\hat{R}}{R^2} \rho(\vec{r}') d\tau'$$

Thus

$$\nabla \cdot \vec{E} = \frac{1}{4\pi\varepsilon_0} \int 4\pi\delta^3(\vec{r} - \vec{r}') \rho(\vec{r}') d\tau' = \frac{1}{\varepsilon_0} \rho(\vec{r})$$

Curl of **E**

point charge : $$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\vec{\mathbf{r}}}$$

$$\boxed{\nabla \times \mathbf{E} = 0}$$

$$\oint \mathbf{E} \cdot d\mathbf{a} = 0$$

General properties

remain true for $\mathbf{E} = \sum_i \mathbf{E}_i$

Poisson's Eq. & Laplace's Eq.

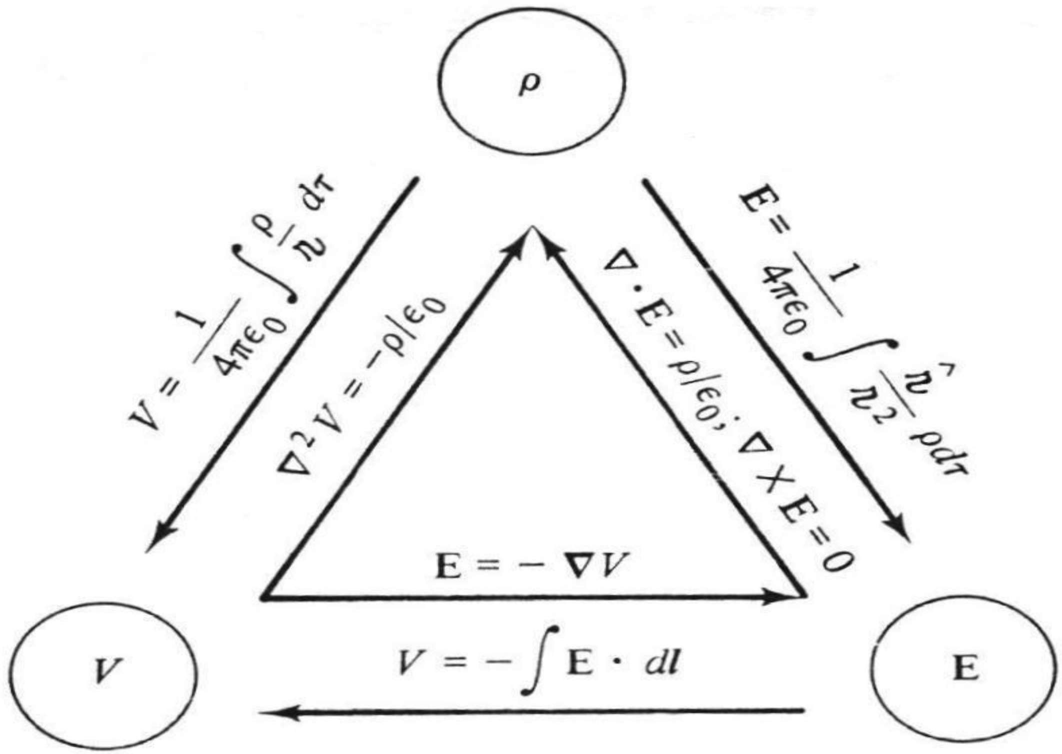
$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} = \nabla \cdot \left(-\nabla V\right) = -\nabla^2 V$$

$$\vec{E} = -\nabla \; V$$

Poisson's Eq. $$\boxed{\nabla^2 V = -\frac{\rho}{\varepsilon_0}}$$

$\rho = 0$ $$\boxed{\nabla^2 V = 0}$$ Laplace's eq.

Electrostatic Boundary Condition



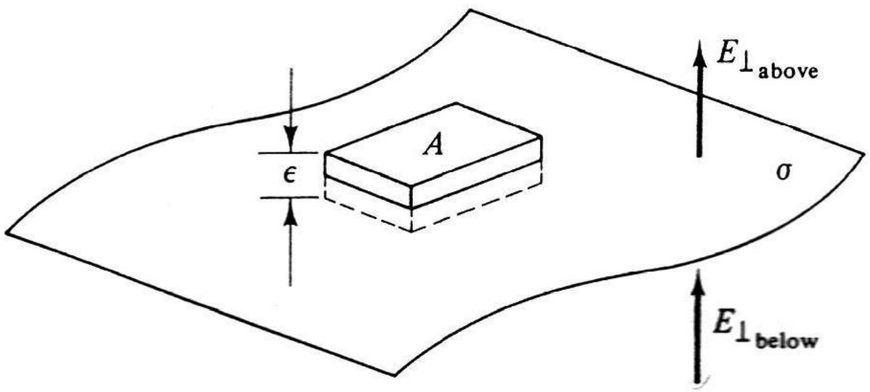
The above equations are differential or integral.

For a unique solution, we need boundary conditions. (e.q. , $V(\infty)=0$)

(boundary value problem. Dynamics: *initial value problem*.)

Boundary condition at surface with charge

$\perp$

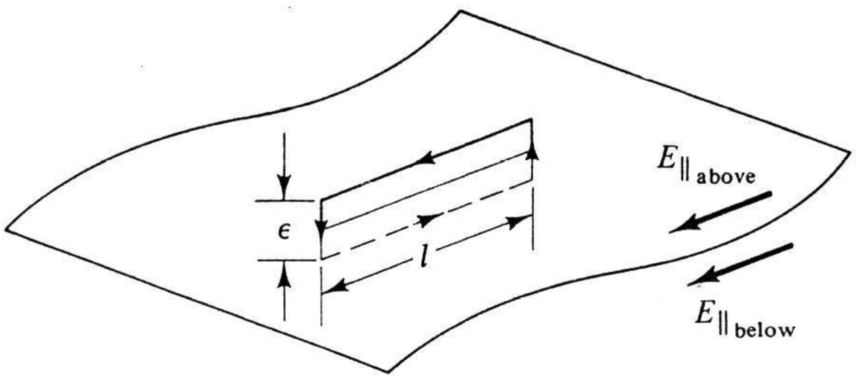


$$\oint_{surface} \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$E_{\perp above} \cdot A - E_{\perp below} \cdot A$$

$$E_{\perp above} - E_{\perp below} = \frac{\sigma}{\varepsilon_0}$$

$\parallel$



$$\oint \vec{E} \cdot d\vec{l} = 0 \qquad \left(\because \nabla \times \vec{E} = 0\right)$$

$$E_{\parallel\, above} = E_{\parallel\, below}$$

$\perp + \parallel$ $$\boxed{\vec{E}_{above} - \vec{E}_{below} = \frac{\sigma}{\varepsilon_0}\hat{n}}$$

- Potential Boundary condition

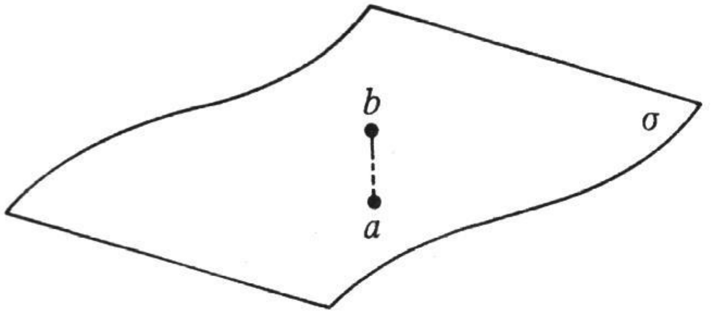


Figure 2.38

$$\because V_{above} - V_{below} = -\int_a^b \vec{E} \cdot d\vec{\ell} \xrightarrow{\overline{ab} \to 0} 0$$

$$\boxed{V_{above} = V_{below}}$$

$$\because \vec{E}_{above} - \vec{E}_{below} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

$$\vec{E} = -\nabla V$$

$$\frac{\partial}{\partial n}V = \nabla V \cdot \hat{n}$$

The Work Done in Moving a charge

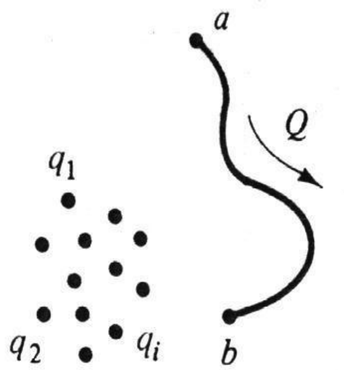
A test charge Q feels a force Q $\vec{E}$

To move this test charge, we have to apply a force $\vec{F} = -Q\vec{E}$

conservative

The total work we do is

$$W = \int_a^b \vec{F} \cdot d\vec{\ell} = -Q\int_a^b \vec{E} \cdot d\vec{\ell} = Q\left[V(b) - V(a)\right]$$



$$\boxed{V(b) - V(a) = \frac{W}{Q}}$$

So, bring a charge from ∞ to P, the work we do is

$$W = Q\left[V(P) - V(\infty)\right] = QV(P)$$

$$V(\infty) = 0$$

The Energy of a Point Charge Distribution

It takes no work to bring in first charges

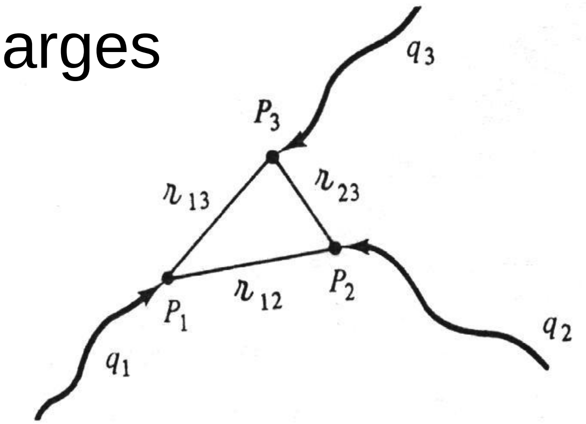


Figure 2.40

$W_1 = 0$ for q_1

Work needed to bring in q_2 is :

$$W_2 = q_2[\frac{1}{4\pi\varepsilon_0}\frac{q_1}{R_{12}}] = \frac{1}{4\pi\varepsilon_0}q_2(\frac{q_1}{R_{12}})$$

Work needed to bring in q_3 is :

$$W_3 = q_3[\frac{1}{4\pi\varepsilon_0}\frac{q_1}{R_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R_{23}}] = \frac{1}{4\pi\varepsilon_0}q_3(\frac{q_1}{R_{13}} + \frac{q_2}{R_{23}})$$

Work needed to bring in q_4 is :

$$W_4 = \frac{1}{4\pi\varepsilon_0}q_4[\frac{q_1}{R_{14}} + \frac{q_2}{R_{24}} + \frac{q_3}{R_{34}}]$$

Total work

W=W$_1$+ W$_2$+ W$_3$ +W$_4$

$$=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{R_{12}}+\frac{q_1q_3}{R_{13}}+\frac{q_2q_3}{R_{23}}+\frac{q_1q_4}{R_{14}}+\frac{q_2q_4}{R_{24}}+\frac{q_3q_4}{R_{34}}\right)$$

$$=\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{q_iq_j}{R_{ij}}$$

$$\underline{\underline{R_{ij}=R_{ji}}} \quad \frac{1}{8\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{q_iq_j}{R_{ij}} \quad =\frac{1}{2}\sum_{i=1}^{n}q_i\left(\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{1}{4\pi\varepsilon_0}\frac{q_j}{R_{ij}}\right)$$

$$\boxed{W=\frac{1}{2}\sum_{i=1}^{n}q_iV\left(P_i\right)} \qquad V\left(P_i\right)=\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{1}{4\pi\varepsilon_0}\frac{q_j}{R_{ij}}$$

Dose not include the first charge

The Energy of a Continuous Charge Distribution

$$\boxed{W = \frac{1}{2}\int \rho V d\tau}$$

$$\delta q = \rho \, d\tau$$

Volume charge density

$$\rho = \varepsilon_0 \nabla \cdot \vec{E} \qquad \nabla \cdot \left(\vec{E}V\right) = \left(\nabla \cdot \vec{E}\right)V + \vec{E} \cdot \left(\nabla V\right)$$

$$\frac{\varepsilon_0}{2}\int\left(\nabla \cdot \vec{E}\right)V d\tau = \frac{\varepsilon_0}{2}\left[\int \nabla \cdot \left(\vec{E}V\right)d\tau + \int \vec{E} \cdot \left(-\nabla V\right)d\tau\right]$$

$\|$ F.T. for $\nabla \cdot$ $\qquad\qquad \|$

$$\oiint_{surface} V\vec{E} \cdot d\vec{a} \qquad\qquad \vec{E}$$

$$= \frac{\varepsilon_0}{2}\left(\oiint_{surface} V\vec{E} \cdot d\vec{a} + \oiint_{volume} E^2 d\tau\right)$$

$$surface \to \infty \quad \Rightarrow \quad \boxed{W = \frac{\varepsilon_0}{2}\int_{all\ space} E^2 d\tau}$$

Example: Find the energy of a uniformly charged spherical shell of total charge q and radius R

Sol.1： $q = 4\pi R^2 \sigma$ $\qquad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

$$W = \frac{1}{2}\int \rho V d\tau = \frac{1}{2}\int \sigma V da = \frac{1}{2}\int \frac{q}{A}\frac{1}{4\pi\varepsilon_0}\frac{q}{R} da = \frac{1}{8\pi\varepsilon_0}\frac{q^2}{R}$$

Sol.2： $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ $\qquad E^2 = \frac{q^2}{(4\pi\varepsilon_0)^2 r^4}$

$$W = \frac{\varepsilon_0}{2}\int E^2 d\tau = \frac{\varepsilon_0}{2}\int_{space}^{outer} \frac{q^2}{(4\pi\varepsilon_o)^2}\frac{1}{r^4}\left(r^2 \sin\theta\, d\theta\, d\varphi\, dr\right)$$

$$= \frac{q^2}{32\pi^2\varepsilon_0}\left[2\cdot 2\pi \int_R^\infty \frac{1}{r^2} dr\right] = \frac{1}{8\pi\varepsilon_0}\frac{q^2}{R}$$

A Dipole in an Electric Field

Although the net force on the dipole from the field is zero, and the center of mass of the dipole does not move, the forces on the charged ends do produce a net torque $\boldsymbol{\tau}$ on the dipole about its center of mass.

The center of mass lies on the line connecting the charged ends, at some distance *x* from one end and a distance *d* -*x* from the other end.

The net torque is:

$$\tau = Fx \sin\theta + F(d-x)\sin\theta = Fd\sin\theta.$$

$$= pE\sin\theta.$$

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on a dipole).}}$$

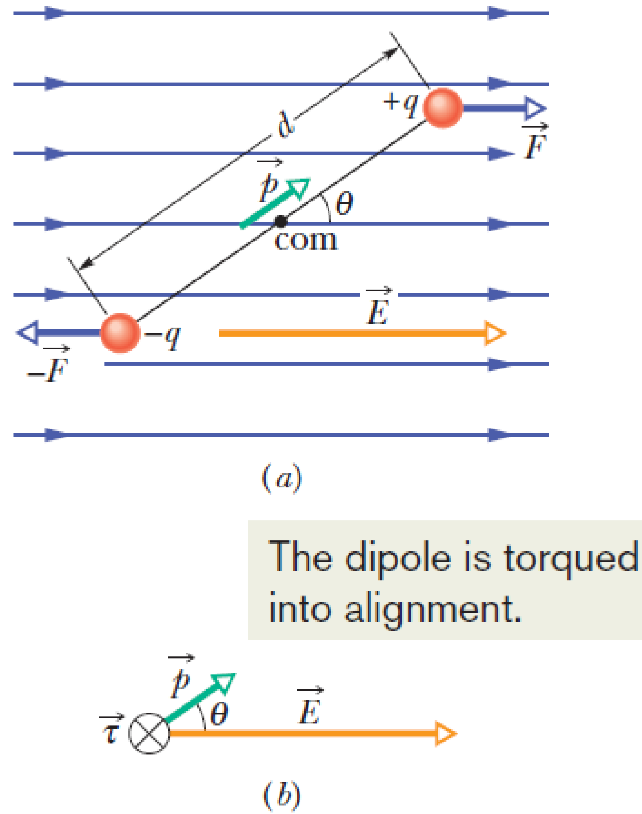


Fig. 22-19 (*a*) An electric dipole in a uniform external electric field $\vec{E}$. Two centers of equal but opposite charge are separated by distance *d*. The line between them represents their rigid connection. (*b*) Field $\vec{E}$ causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol ⊗.

A Dipole in an Electric Field: Potential Energy

Potential energy can be associated with the orientation of an electric dipole in an electric field.

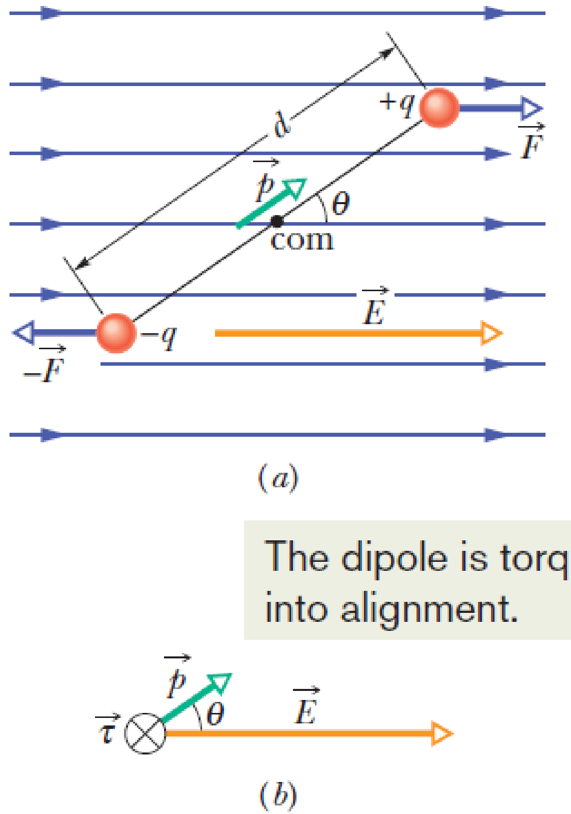
The dipole has its least potential energy when it is in its equilibrium orientation, which is when its moment ***p*** is lined up with the field ***E*.**

The expression for the potential energy of an electric dipole in an external electric field is simplest if we choose the potential energy to be zero when the angle θ (Fig.22-19) is 90°.

The potential energy U of the dipole at any other value of θ can be found by calculating the work W done by the field on the dipole when the dipole is rotated to that value of θ from 90°.

$$U = -W = -\int_{90^\circ}^{\theta} \tau \, d\theta = \int_{90^\circ}^{\theta} pE \sin\theta \, d\theta = -pE\cos\theta.$$

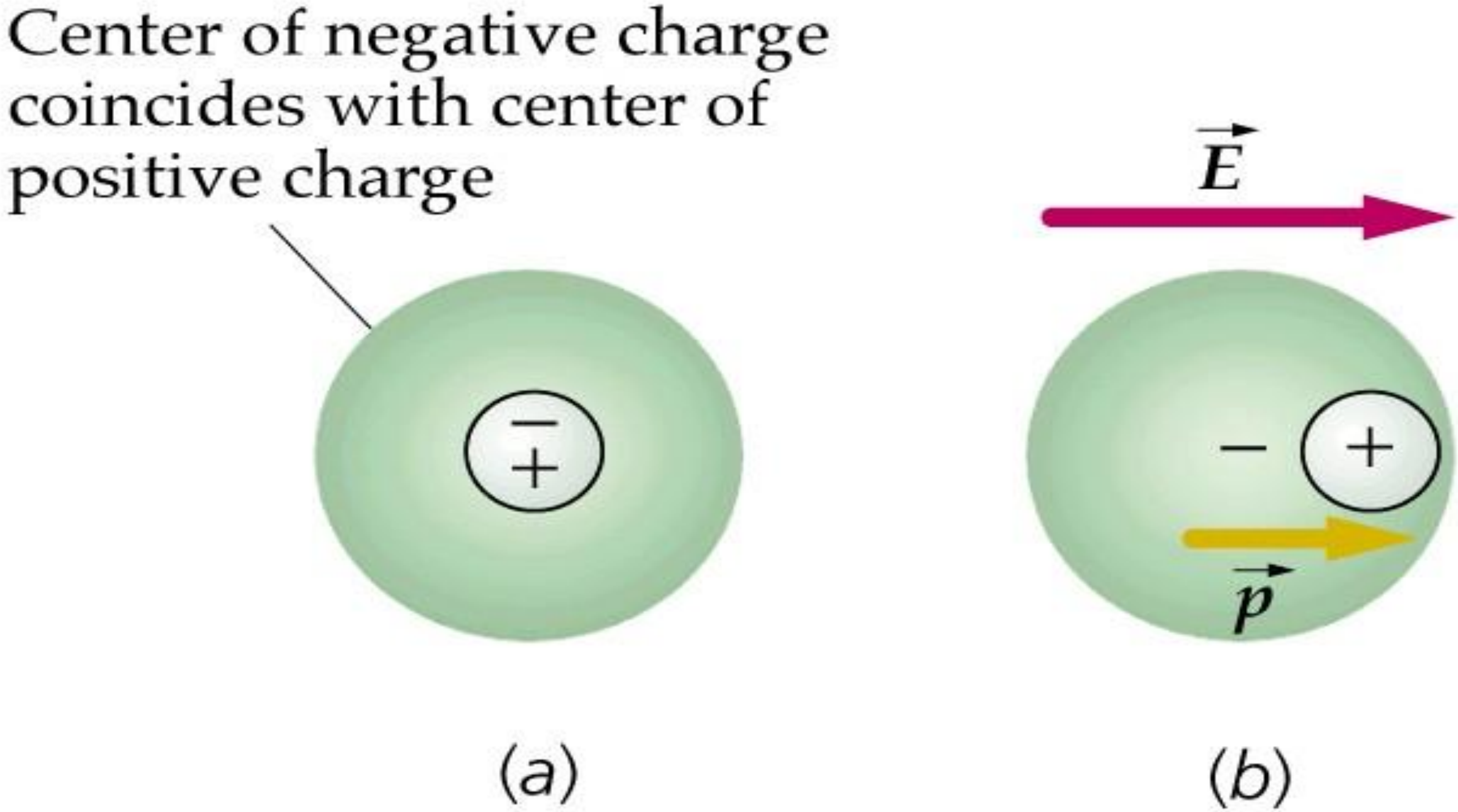
$$U = -\vec{p} \cdot \vec{E} \quad \text{(potential energy of a dipole).}$$



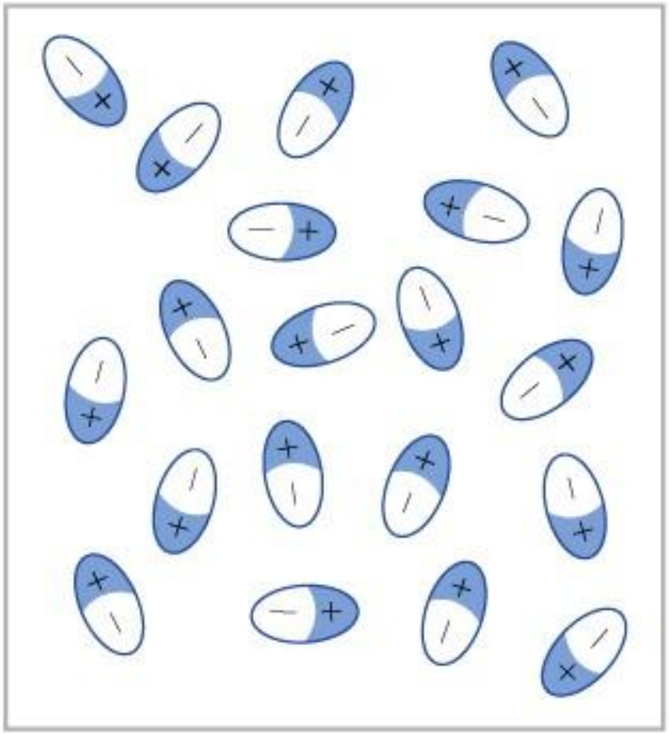
The dipole is torqued into alignment.

Fig. 22-19 (*a*) An electric dipole in a uniform external electric field $\vec{E}$. Two centers of equal but opposite charge are separated by distance d. The line between them represents their rigid connection. (*b*) Field $\vec{E}$ causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol ⊗.

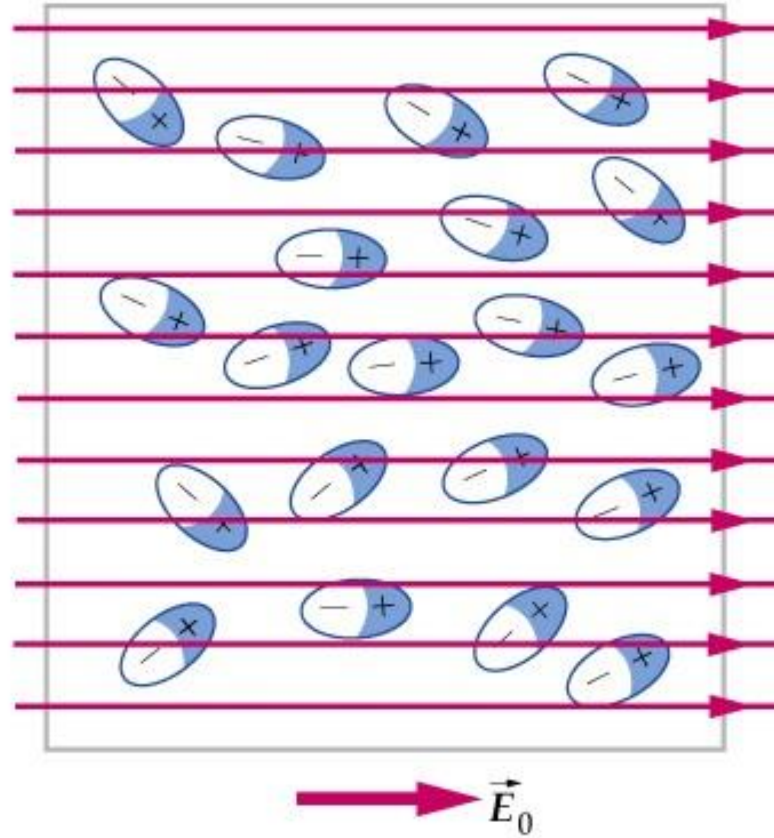
Electric polarization



$$\mathbf{p} = \alpha \mathbf{E}$$



(a)



(b)

$$\mathbf{p} = \alpha \mathbf{E}$$

The Electric Displacement

Total charge $\rho = \rho_f + \rho_b$

Free charge (at our disposal) ρ_f

Bound charge (induced, comes along) ρ_b

Electric displacement (auxiliary field) $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$

$\nabla \cdot \mathbf{D} = \rho_f$ $\oint \mathbf{D} \cdot d\mathbf{a} = Q_{f,enclosed}$ But in general $\nabla \times \mathbf{D} \neq 0$

Boundary conditions $D^{\perp}_{above} - D^{\perp}_{below} = \sigma_f$ $D^{\parallel}_{above} - D^{\parallel}_{below} = P^{\parallel}_{above} - P^{\parallel}_{below}$

Future Scope and relevance to industry

https://ieeexplore.ieee.org/document/7368644
https://link.springer.com/article/10.1007/s12274-017-1842-6
https://www.nasa.gov/mission_pages/station/research/experiments/2325.html